SELECTED H.O.T.S. QUESTIONS
FROM CBSE 2026 EXAMINATIONS

Unit I - Relations & Functions

Relations and Functions; Inverse Trig. Functions

01. A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1,1), (2,2), (1,2)\}$ is
(a) Reflexive only (b) Reflexive and Transitive
(c) Symmetric and Transitive (d) Transitive only

02. A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1,2), (2,1), (2,2)\}$ is
(a) Reflexive only (b) Reflexive and Transitive
(c) Symmetric an Transitive (d) Symmetric only

03. A relation R on set $A = \{1, 2, 3\}$ is defined as $R = \{(1,3), (3,3), (1,1), (2,2), (3,1)\}$ is
(a) only reflexive and symmetric (b) reflexive only
(c) only reflexive and transitive (d) reflexive, symmetric and transitive

Direction : *Questions given below are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

04. **Assertion (A) :** A relation R on the set $\{1, 2, 3\}$ defined as $R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$ is an equivalence relation.
Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.

05. **Assertion (A) :** A function $f: N \to N$ given by $f(x) = x^3 + 2,\ \forall\ x \in N$ is one-one but not onto.
Reason (R) : Since $\forall\ y \in N$ (Codomain), there does not exist $x = (y-2)^{1/3}$ in N (Domain) such that $f(x) = x^3 + 2 = y$.

06. Check whether $f: Z \times Z \to Z \times Z$ (where Z is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.

07. A relation R is defined on Z, the set of integers, as $R = \{(x, y) : |x - y|$ is divisible by a prime number 'p', $x, y \in Z\}$. Check whether R is an equivalence relation or not.

08. Let n be a fixed positive integer. A relation R is defined in set Z such that $R = \{(x, y) :\ (x - y)$ is divisible by $n;\ x, y \in Z\}$. Determine if R is an equivalence relation.

09. A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1,1), (3,3), (1,2)\}$. Is R a symmetric relation? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$.

10. Show that $f: \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \dfrac{x}{\sqrt{1 + x^2}}$ is one-one but not onto.

11. Show that a function $f: \mathbb{R}_+ \to A \subset \mathbb{R}$, defined as $f(x) = 4x^2 + 12x + 15$ is one-one. Find set A so that f is onto where $\mathbb{R}_+ = [0, \infty)$. Also, find if there exists $a \in \mathbb{R}_+$, such that $f(a) = 7$. Justify.

12. Show that $f: \mathbb{R}_+ \to [-5, \infty)$ given by $f(x) = 4x^2 + 4x - 5$ is both one-one and onto where $\mathbb{R}_+ = [0, \infty)$. Also, find $p \in \mathbb{R}_+$, such that $f(p) = 3$.

13. A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition.
7 students out of 80 are selected for the project and are categorized into two sets such that :
Girl students belong to Set $A = \{G_1, G_2, G_3, G_4\}$,
Boy students belong to Set $B = \{B_1, B_2, B_3\}$.
Based on the above information, answer the following questions.
(i) How many relations are possible from Set A $\to$ Set B?
(ii) Let R be a relation from $A \to B$ such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$.
Is R an injective function? Justify your answer.
(iii) (a) Let the relation R from $A \to A$ be such that $R = \{(x, y) : x,\ y \in A$, x and y are students from the same colony in the city$\}$. Verify if R is an equivalence relation.

OR

(iii) (b) Verify if any function $f : B \to A$ is bijective. Give reason to support your answer.

14. If $2\cos^{-1} x = y$, then
(a) $0 \le y \le \pi$ (b) $-\pi \le y \le \pi$ (c) $0 \le y \le 2\pi$ (d) $-\pi \le y \le 0$

15. If $\tan^{-1} x = 3y$, then
(a) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (b) $-\frac{3\pi}{2} < y < \frac{3\pi}{2}$ (c) $-\frac{\pi}{6} < y < \frac{\pi}{6}$ (d) $-\frac{\pi}{6} \le y \le \frac{\pi}{6}$

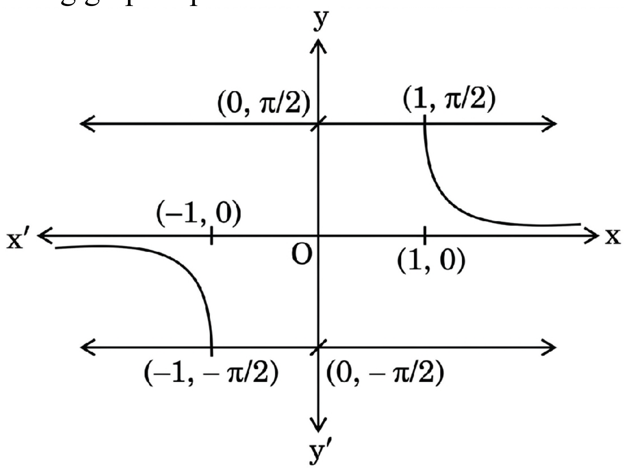
16. If $\sin^{-1} x + \pi = y$, then
(a) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (b) $-\frac{3\pi}{2} \le y \le -\frac{\pi}{2}$ (c) $\frac{\pi}{2} \le y \le \frac{3\pi}{2}$ (d) $0 \le y \le \pi$

17. The domain of $f(x) = \cos^{-1}(2x - 5)$ is
(a) $[-1, 1]$ (b) $[4, 6]$ (c) $[-7, -3]$ (d) $[2, 3]$

18. The domain of $\sin^{-1}(1 - 2x)$ is
(a) $[-1, 1]$ (b) $[-1, 3]$ (c) $[-2, 2]$ (d) $[0, 1]$

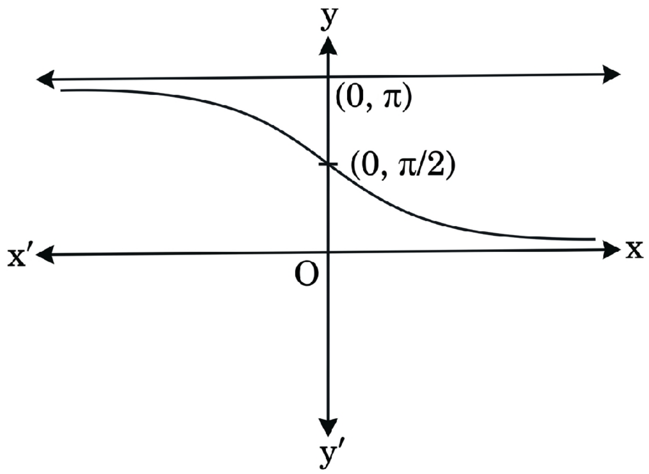
19. The domain of $\cos^{-1}(4x + 1)$ is
(a) $[-1, 1]$ (b) $[-3, 5]$ (c) $[-4, 4]$ (d) $\left[-\frac{1}{2}, 0\right]$

20. The following graph represents



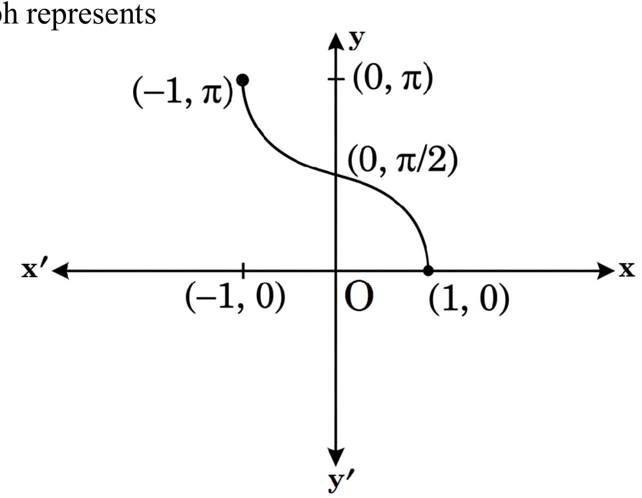
(a) $y = \cos^{-1} x$ (b) $y = \sec^{-1} x$ (c) $y = \tan^{-1} x$ (d) $y = \text{cosec}^{-1} x$

21. The following graph represents



(a) $y = \sec^{-1} x$ (b) $y = \cot^{-1} x$ (c) $y = \tan^{-1} x$ (d) $y = \operatorname{cosec}^{-1} x$

22. The following graph represents



(a) $y = \cos^{-1} x$ (b) $y = \sec^{-1} x$ (c) $y = \sin^{-1} x$ (d) $y = \tan^{-1} x$

23. For the inverse trigonometric functions, which of the following Principal Value Branch is **not** correctly defined?

(a) $\tan^{-1} : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $\sec^{-1} : \mathbb{R} - (-1, 1) \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$

(c) $\cot^{-1} : \mathbb{R} \to (0, \pi)$ (d) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

24. Simplify : $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$, $0 < x < \frac{\pi}{4}$.

25. Find the domain of $g(x) = \cos^{-1}(x^2 - 1)$. Hence, find the value of x for which $g(x) = \frac{\pi}{3}$. Also, write the range of $\cos^{-1} x$ other than its principal branch.

26. Find the domain of $q(x) = \cos^{-1}(4x^2 - 3)$. Hence, find the value of x for which $q(x) = 0$. Also, write the range of $3q(x) - \pi$.

27. Find the domain of $p(x) = \sin^{-1}(1 - 2x^2)$. Hence, find the value of x for which $p(x) = \frac{\pi}{6}$. Also, write the range of $2p(x) + \frac{\pi}{2}$.

28. Find the value of $\sin\left[\cot^{-1} \sqrt{2}\left(\cos(\tan^{-1} 1)\right)\right]$.

29. Evaluate $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)+\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$.

30. Simplify : $\cot^{-1}\sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$, $x\in\left(0,\frac{\pi}{2}\right)$.

31. Simplify : $\sin^{-1}\sqrt{\frac{1+\cos 2x}{2}}$, $0<x<\frac{\pi}{2}$.

32. Evaluate $\sin\left[\tan^{-1}\tan\left(\frac{3\pi}{4}\right)\right]$.

33. Evaluate $\sin\left[\cos^{-1}\cos\left(\frac{7\pi}{6}\right)\right]$.

34. Evaluate $\tan\left[\cos^{-1}\left(\tan\frac{3\pi}{4}\right)\right]$.

Unit II - *Algebra*

Matrices; Determinants

01. Which of the following cannot be the order of a row-matrix?
(a) 2×1 (b) 1×2 (c) 1×1 (d) $1\times n$

02. Which of the following cannot be an order of a column-matrix?
(a) 1×2 (b) 2×1 (c) 1×1 (d) $m\times1$, where $m\in N$

03. Which of the following properties is/are true for two matrices of suitable orders?
(i) $(A+B)'=A'+B'$ (ii) $(A-B)'=B'-A'$
(iii) $(AB)'=A'B'$ (iv) $(kAB)'=kB'A'$ (k is a scalar)
(a) (i) only (b) (i), (ii) and (iii) (c) (i) and (ii) (d) (i) and (iv)

04. If $\Delta_1=\begin{vmatrix}1&0&0\\0&2&0\\0&0&3\end{vmatrix}$ and $\Delta_2=\begin{vmatrix}0&2&0\\1&0&0\\0&0&6\end{vmatrix}$, then
(a) $\Delta_1=2\Delta_2$ (b) $\Delta_2=-2\Delta_1$ (c) $\Delta_1=\Delta_2$ (d) $\Delta_2=-\Delta_1$

05. One of the values of x for which $\begin{vmatrix}\cos x&\sin x\\-\cos x&\sin x\end{vmatrix}=1$ is
(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

06. If A and B are skew symmetric matrices or same order, then which of the following matrices is also skew symmetric?
(a) AB (b) $AB+BA$ (c) $(A+B)^2$ (d) $A-B$

07. If A and B are symmetric matrices of same order, then which of the following matrices is a skew-symmetric matrix?
(a) A^2+B^2 (b) A^2-B^2 (c) $AB+BA$ (d) $AB-BA$

08. If $A=\left[a_{ij}\right]_{3\times3}$ is a scalar matrix, then which of the following must be true?
(a) A must be a symmetric matrix (b) A must be a skew-symmetric matrix
(c) A must be an identity matrix (d) A must be a null matrix

09. If A and B are symmetric matrices of same order, then $(AB-BA)$ is
(a) zero matrix (b) identity matrix (c) symmetric matrix (d) skew symmetric matrix

10. If A and B are square matrices of same order, then which of the following statements is/are always true?
(i) $(A+B)(A-B)=A^2-B^2$
(ii) $AB=BA$

(iii) $(A+B)^2 = A^2 + AB + BA + B^2$

(iv) $AB = O \Rightarrow A = O$ or $B = O$

(a) Only (i) and (iii) (b) Only (ii) and (iii)

(c) Only (iii) (d) Only (iii) and (iv)

11. If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a+b+c$ is

(a) 2 (b) 6 (c) 4 (d) 0

12. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ and $A + A' = I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

13. For an invertible square matrix A, $(3A)^{-1} =$

(a) $3A^{-1}$ (b) $9A^{-1}$ (c) $\frac{1}{3}A^{-1}$ (d) $\frac{1}{9}A^{-1}$

14. If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is

(a) 4 (b) 5 (c) –4 (d) 9

15. If $A = \begin{bmatrix} \frac{1}{2}\cos x & -\sin x \\ \sin x & \frac{1}{2}\cos x \end{bmatrix}$ and $A + A' = I$, then value of $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) 0 (d) $\frac{-\pi}{2}$

16. If $A = \begin{bmatrix} \tan x & \cot x \\ -\cot x & \tan x \end{bmatrix}$ and $A + A' = 2I$, then value of $x \in \left[0, \frac{\pi}{2}\right]$ is

(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

17. If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x+y)$ is

(a) –1 (b) 1 (c) $\frac{5}{3}$ (d) 7

18. If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an

(a) symmetric matrix (b) skew-symmetric matrix

(c) null matrix (d) identity matrix

19. If $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$, then order of A must be

(a) 3×1 (b) 1×3 (c) 1×1 (d) 3×3

20. If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be

(a) 7 (b) 5 (c) –7 (d) –1

21. If $B(\text{adj.}B) = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$, then the value of $\det(B^{-1}) =$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) 3 (d) 9

22. If a matrix B is such that $B\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, then the order of matrix B is

(a) 1×3 (b) 3×1 (c) 3×3 (d) 1×1

23. If $A(\text{adj.}A) = \begin{bmatrix} 2026 & 0 & 0 \\ 0 & 2026 & 0 \\ 0 & 0 & 2026 \end{bmatrix}$, then the value of $|\text{adj.}A|$ is equal to

(a) 2026 (b) $(2026)^{-1}$ (c) $(2026)^{-2}$ (d) $(2026)^2$

24. If a matrix X is such that $\begin{bmatrix} 2 & 1 \end{bmatrix}X = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$, then the order of matrix X is

(a) 3×1 (b) 2×3 (c) 1×2 (d) 1×3

25. If $A(\text{adj.}A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|2A|$ is

(a) 6 (b) 54 (c) 12 (d) 24

26. Let $A = [a_{ij}]$ be a 2×2 matrix whose elements are given by $a_{ij} = \frac{(i+2j)^2}{3}$. Then A' is

(a) $\begin{bmatrix} 12 & \frac{25}{3} \\ \frac{16}{3} & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 12 & \frac{16}{3} \\ \frac{25}{3} & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & \frac{25}{3} \\ \frac{16}{3} & 12 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$

27. If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is

(a) $\frac{4}{7}$ (b) $-\frac{3}{7}$ (c) 4 (d) –4

28. If A is a non-singular matrix, then which of the following is **not** true?

(a) adj. A is singular (b) $(\text{adj.}A)^{-1} = (\text{adj.}A^{-1})$

(c) $|A| \neq 0$ (d) A^{-1} exists

29. If the area of ΔABC with vertices A(3, 1), B(–2, 1) and C(0, k) is 5 sq. units, then values of k are

(a) 3, 1 (b) –1, 3 (c) –1, 2 (d) 0, 2

30. For any square matrix A with real entries, if $A + A'$ is a symmetric matrix, then

(a) $(A - A')$ cannot be a skew symmetric matrix

(b) $(A - A')$ is a skew symmetric matrix

(c) A is always a symmetric matrix

(d) A is always a skew symmetric matrix

31. A matrix $B=[b_{ij}]_{m\times m}$ is said to be a diagonal matrix, if

(a) $b_{ij}=0$, when $i=j$ (b) $b_{ij}=1$, when $i=j$

(c) $b_{ij}=1$, when $i\neq j$ (d) $b_{ij}=0$, when $i\neq j$

Direction : *Question given below is Assertion (A) and Reason (R) based question carrying* ***1 mark****. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

32. **Assertion (A) :** If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB=BA$.
Reason (R) : Product of two diagonal matrices of same order is commutative.

33. If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x, where $x\in\left[0,\dfrac{\pi}{2}\right]$.

34. Obtain the value of $\Delta=\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z. Further, if $\Delta=0$ and x, y, z are non-zero real numbers, prove that $x^{-1}+y^{-1}+z^{-1}=-1$.

35. On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of ₹1000 and ₹500 were given to the lucky draw winners.

A total of 60 vouchers were given on the day. The number of ₹1000 vouchers added to 3 times the number of ₹500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.

36. A man goes to buy fruits from the market. The shopkeeper informs him that 4 apples, 3 oranges and 2 bananas cost ₹60; 2 apples, 4 oranges and 6 bananas cost ₹90; whereas 6 apples, 2 oranges and 3 bananas cost ₹70. Using matrix method, find the cost of one fruit of each kind.

37. Find cost (per kg) of each fertilizer A, B and C that the farmer needs to buy such that 1 kg each of fertilizer A and C added to 2 kg of B costs him ₹400. Also, cost of each kg of fertilizer B and C added together is equal to cost of 1 kg of fertilizer A. However, cost of 3 kg of fertilizer B

added to ₹200 is the same as cost of 1 kg of fertilizer A and C together. Use matrix method to find the solution.

38. Three students A, B and C go to a book-store to buy art books, story books and puzzle solving books. A buys one of each type of book for a total of ₹21. B buys 4 art books, 3 story books and 2 puzzle solving books for ₹60. C buys 6 art books, 2 story books and 3 puzzle solving books and pays ₹10 more than B. Use matrix method to find the cost of each type of book.

39. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, then compute $A^2-4A-5I$.

40. If $A=\begin{bmatrix}3 & 2 & 0\\ 1 & 4 & 0\\ 0 & 0 & 5\end{bmatrix}$, then compute $A^2-7A+10I$.

41. If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, then compute $A^2-5A+4I$.

42. A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length.
Based on the given information, answer the following questions.
(i) Write the equations representing the various dimensions and express them as the matrix equation $AX=B$.
(ii) Find if A^{-1} exists. Justify your answer.
(iii) (a) Find A^{-1}.

OR

(iii) (b) Find A^2+7I.

Unit III - Calculus

Continuity and Differentiability; Applications of Derivatives; Integrals; Applications of Integrals; Differential Equations

01. If $e^{-x}+e^{-y}=2$, then $\frac{dy}{dx}$ is
(a) e^{x-y} (b) e^{y-x} (c) $-e^{x-y}$ (d) $-e^{y-x}$

02. If $e^{x+y}=3x$, then $\frac{dy}{dx}$ is
(a) $\frac{3}{e^{x+y}}$ (b) $\frac{1}{e^{x+y}}$ (c) $\frac{1-e^{x+y}}{e^{x+y}}$ (d) $\frac{3-e^{x+y}}{e^{x+y}}$

03. If $x+y=xy$, then $\frac{dy}{dx}$ is
(a) $\frac{y}{x-1}$ (b) $\frac{1}{x-1}$ (c) $\frac{y-1}{x-1}$ (d) $\frac{1-y}{x-1}$

04. The value of k for which the function $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0 \\ k(x+1), & x=0 \end{cases}$ is a continuous function, is

(a) $\frac{1}{4}$ (b) 2 (c) $\frac{1}{2}$ (d) 0

05. If $\sin^{-1}x=y$, then $\frac{dy}{dx}$ is

(a) $\cos^{-1}x$ (b) $\cos y$ (c) $\frac{1}{1-x^2}$ (d) $\sec y$

06. If $\tan^{-1}x=y$, then $\frac{dy}{dx}$ is equal to

(a) $(\sec^{-1}x)^2$ (b) $\sec^2 y$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\cos^2 y$

07. If $2\cos^{-1}x=y$, then $\frac{dy}{dx}$ is

(a) $-2\sin^{-1}x$ (b) $-\frac{1}{2}\sin\frac{y}{2}$ (c) $\frac{-1}{\sqrt{1-2x^2}}$ (d) $-2\operatorname{cosec}\frac{y}{2}$

08. Differential of e^{e^x} with respect to x is

(a) $\log x$ (b) e^{e^x} (c) $e^x e^{e^x}$ (d) $(e^x)^2$

09. The greatest integer function, $f(x)=[x],\ 0<x<3$ is **not** differentiable at how many points?

(a) At only one point (b) At only two points
(c) At no point (d) At three points

10. If $f(x)=\begin{cases} \frac{x^2-4x-5}{x+1}, & x\neq -1 \\ k, & x=-1 \end{cases}$ is continuous at $x=-1$, then the value of k is

(a) Any real value (b) 6 (c) –1 (d) –6

11. Derivative of $\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right),\ -\frac{\pi}{4}<x<\frac{\pi}{4}$ with respect to x is

(a) –1 (b) 1 (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

12. Check whether function $f(x)$ defined as $f(x)=\begin{cases} \frac{|x-3|}{2(x-3)}, & x<3 \\ \frac{x-6}{6}, & x\geq 3 \end{cases}$ is continuous at $x=3$ or not.

13. If $\sqrt{3}(x^2+y^2)=4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$.

14. If $x=\sin t-\cos t,\ y=\sin t\cos t$, find $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$.

15. If $x=e^{\sin^{-1}t},\ y=e^{\cos^{-1}t}$, then find $\frac{dy}{dx}$ at $t=\frac{1}{\sqrt{2}}$.

16. If $x=t+\frac{1}{t}$ and $y=t-\frac{1}{t}$, find $\frac{dy}{dx}$ at $t=2$.

17. If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.

18. If $x = e^{t+\frac{1}{t}}$, $y = e^{t-\frac{1}{t}}$, then find $\frac{dy}{dx}$ at $t = -2$.

19. If $x = e^{\sin^{-1} t}$, $y = e^{\cos^{-1} t}$, then find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$.

20. Show that the function $f(x) = \begin{cases} \dfrac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

21. Find whether the function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.

22. If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.

23. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ with respect to $\cos^{-1} x^2$.

24. Differentiate x^x with respect to $x \log x$.

25. If $y\sqrt{x^2+1} = \log\left[\sqrt{x^2+1} - x\right]$, show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$.

26. Find the differential of $x^{\cot x} + \dfrac{2x^2 - 3}{2x^2 - x + 2}$ with respect to x.

27. If $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$, find $\frac{dy}{dx}$ and prove that $\frac{d^2y}{dx^2} = \dfrac{-\text{cosec}^3 2t.\text{cosec}\, t}{3}$.

28. Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.

The equation of one such track is given as follows:

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \leq x < 3 \\ x^2 + 40, & x \geq 3 \end{cases}$$

Based on the given information, answer the following questions.

(i) Find $f'(x)$ for $0 < x < 3$.

(ii) Find $f'(4)$.

(iii) (a) Test for continuity of $f(x)$ at $x = 3$.

OR

(iii) (b) Test for differentiability of $f(x)$ at $x = 3$.

29. The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is

(a) -16 (b) -9 (c) 0 (d) 16

30. The absolute maximum value of $f(x) = x^2 + 1$ in $[-5, 2]$ is
(a) 26 (b) 1 (c) 5 (d) 2

31. The least value of $f(x) = e^{-x}$ in $[0, 3]$ is
(a) e^{-3} (b) -1 (c) 1 (d) $-e^3$

32. Absolute minimum value of $f(x) = (x-2)^2 + 5$ in the interval $[-3, 2]$ is
(a) –3 (b) 2 (c) 5 (d) 30

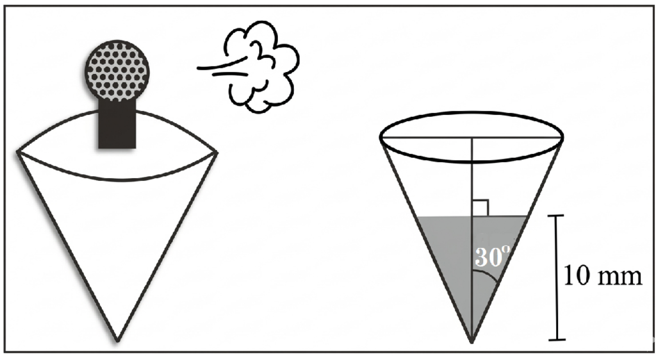
33. For $f(x) = x + \frac{1}{x}$ $(x \neq 0)$
(a) local maximum value is 2
(b) local minimum value is –2
(c) local maximum value is –2
(d) local minimum value $<$ local maximum value

34. The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is
(a) 400π cm^3/cm (b) 100π cm^3/cm (c) 50π cm^3/cm (d) 25π cm^3/cm

35. If the distance travelled by a particle in t seconds is given by $S = 72t + 3t^2 - t^3$, then time taken by the particle to come to rest is
(a) 4 seconds (b) 6 seconds (c) 3 seconds (d) 0 seconds

36. A cylindrical tank is being filled with sand at a rate of 314 m^3/h. If the radius of the tank is 10 m, then the height of sand in the tank increases at the rate of
(a) 1.1 m/h (b) 1 m/h (c) π m/h (d) $\frac{\pi}{2}$ m/h

37. A room freshener bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of 1 mm^3/min. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm, if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$.



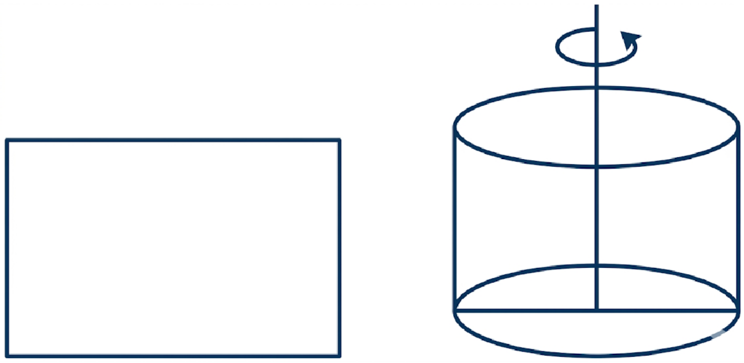
38. If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.

39. Find the sub intervals in which $f(x) = \cot^{-1}(\sin x + \cos x)$, $x \in (0, \pi)$ is increasing and decreasing.

40. Find the sub intervals of $(0, \pi)$ in which $f(x) = \tan^{-1}(\sin x - \cos x)$ is increasing and decreasing.

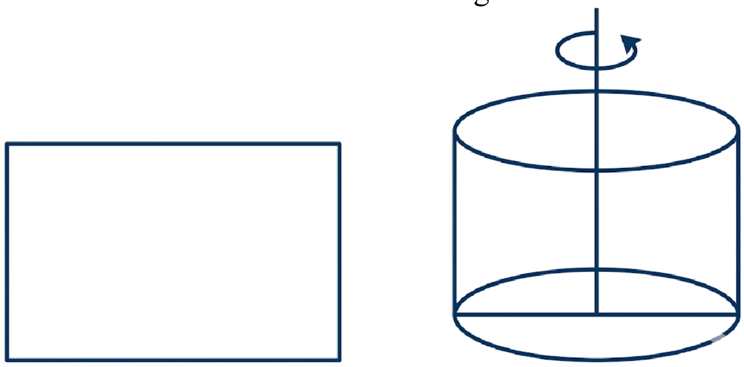
41. Find the sub intervals of $\left(0, \frac{\pi}{2}\right)$ in which $f(x) = \log(\sin x + \cos x)$ is increasing and decreasing.

42. A rectangle of perimeter 36 cm is revolved around one of its sides to sweep out a cylinder of maximum volume.



Find the dimensions of the rectangle.

43. A rectangle of perimeter 30 cm is revolved along one of its sides to sweep out a cylinder of maximum volume. Find the dimensions of the rectangle.



44. Find the sub-interval (s) of $\left(0, \frac{\pi}{2}\right)$ in which $f(x) = \tan x - 4x$ is increasing.

45. Find the sub-interval of $(0, \infty)$ in which $f(x) = x^2 e^{-x}$ is increasing.

46. Find the sub-interval of $(0, \infty)$ in which $f(x) = x \log x$ is increasing.

47. Find the values of x for which $f(x) = x^x$, $x > 0$ is increasing.

48. Find the interval (s) in which the function $f(x) = \frac{x}{\log x}$, where $x \in (0, 1) \cup (1, \infty)$, is increasing.

49. Find the interval (s) for which the function $f(x) = \frac{x}{3} + \frac{3}{x}$, $x \neq 0$ is increasing.

50. Determine the values of x for which $f(x) = \frac{x-3}{x+1}$, $x \neq -1$ is an increasing function.

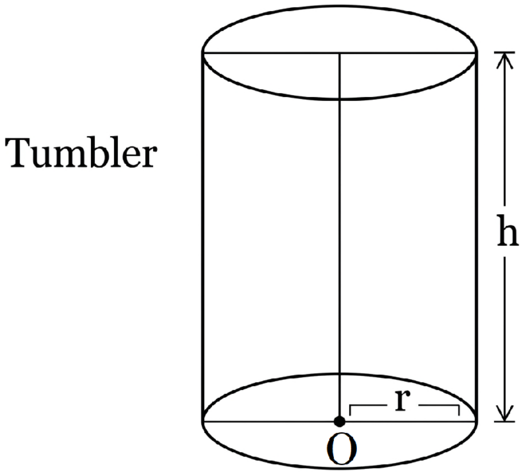
51. A spherical balloon loses its volume due to escape of air from it in such a way that decrease of volume at any instant is proportional to its surface area. Show that the radius is decreasing at a constant rate.

52. Determine the interval (s) in which $f(x) = 5x^{3/2} - 3x^{5/2}$, $x > 0$ is increasing.

53. A thin metallic wire in the shape of a circular ring has its enclosed area increasing at a uniform rate when heated. Show that the rate of change of circumference varies inversely as the radius.

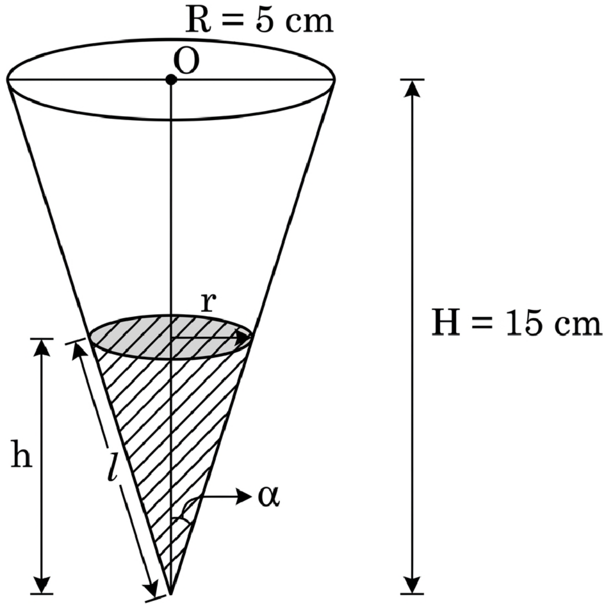
54. The volume of a wooden block in the shape of a cube increases at a constant rate as the air becomes moist during the rainy season. Show that the rate of change of its surface area varies inversely as the length of edge of the cube.

55. A company produce cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.



Based on the above information, answer the following questions.
If for a tumbler, V is its volume, h the height and r the radius of the circular base, then
(i) differentiate its volume with respect to radius of the base, where the surface area is constant.
(ii) if the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base.

56. At a birthday party, children are being served orange juice in conical cups, as shown in the figure.

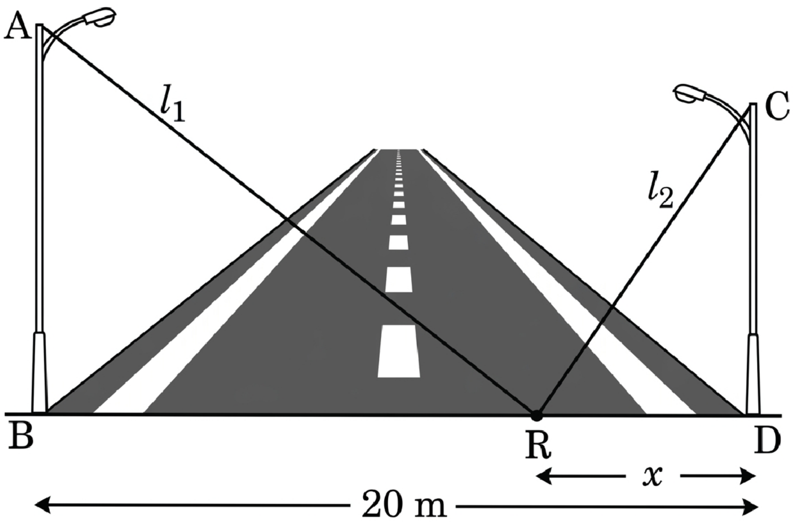


Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1 \text{ cm}^3/\text{s}$.
Based on the above information, answer the following questions.
(i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α.
(ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep?
(iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing?

OR

(iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing.

57. Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.

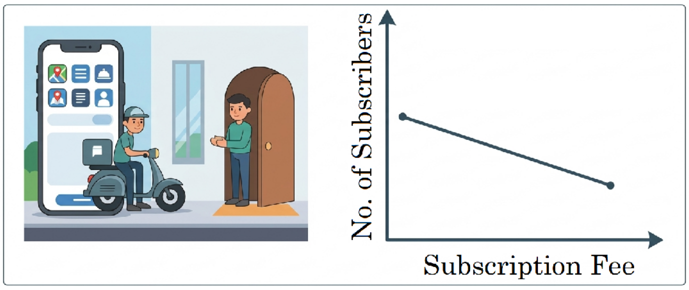


Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.
Based on the above information, answer the following questions.
(i) Express $p(x) = l_1 + l_2$ in terms of x.
(ii) Find $p'(x)$.
(iii) (a) Find the value of x for which $l_1^2 + l_2^2$ is minimum.

OR

(iii) (b) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum?

58. An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹300 per subscriber for unlimited free deliveries.



The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹x.
Based on the given information, answer the following questions.
(i) How many subscribers will discontinue after an increase of ₹x in the annual fee?
(ii) If R(x) denotes the total revenue collected after the increase of ₹x in subscription fee, express R(x) as a function of x.
(iii) (a) Find the value of x for which R(x) is maximum.

OR

(iii) (b) Find the sub-intervals of (0, 5000) in which R(x) is increasing and decreasing.

59. If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log|b^2 + c^2x^2| + K$, then the value of A is

(a) $3a$ (b) $\frac{3a}{2b^2}$ (c) $\frac{3a}{b^2c^2}$ (d) $\frac{3a}{2c^2}$

60. The value of $\int_{-1}^{1} \frac{x^3}{x^2+2|x|+1} dx$ is

(a) 0 (b) $\log 2$ (c) $2\log 2$ (d) $\frac{1}{2}\log 2$

61. If $\int_{0}^{2a} \frac{1}{1+4x^2} dx = \frac{\pi}{6}$, then the value of a is

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

62. $\int \frac{dx}{2^x + 2^{-x}}$ is equal to

(a) $\tan^{-1}(2^x) + C$ (b) $\tan^{-1}(2^{-x}) + C$ (c) $\frac{\tan^{-1}(2^x)}{\log 2} + C$ (d) $(\log 2)\tan^{-1}(2^x) + C$

63. $\int_{-1}^{1} (1-|x|) dx$ is equal to

(a) $2\int_{0}^{1}(1+x)dx$ (b) $2\int_{-1}^{0}(1+x)dx$ (c) 0 (d) $2\int_{-1}^{0}(1-x)dx$

64. $\int \frac{dx}{\sqrt{e^{-2x}-1}}$ is equal to

(a) $\sin^{-1} e^{-x} + C$ (b) $\log\left|e^{-x} + \sqrt{e^{-2x}-1}\right| + C$

(c) $\sin^{-1} e^{x} + C$ (d) $\log\left|e^{-x} - \sqrt{e^{-2x}-1}\right| + C$

65. $\int_{0}^{1} \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is

(a) e (b) $\frac{\pi}{4}$ (c) 0 (d) $-\frac{\pi}{4}$

66. The value of $\int_{0}^{1} \frac{1}{3x-4} dx$ is

(a) $\frac{1}{3}\log 4$ (b) $-\frac{1}{3}\log 4$ (c) $\log(-4)$ (d) $\log 4$

67. If $\int_{0}^{1} (6x^2 - 4x + k)dx = 0$, then the value of k is

(a) 1 (b) 0 (c) 2 (d) –1

68. Evaluate $\int_{0}^{2} \frac{1}{\sqrt{x^2+2x+3}} dx$.

69. Evaluate $\int_{0}^{1} \frac{x\tan^{-1}x}{(1+x^2)^{3/2}} dx$.

70. Find $\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$.

71. Evaluate $\int_{0}^{\pi} \frac{\sin^{2026}x}{\sin^{2026}x + \cos^{2026}x} dx$.

72. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{2^x+1} dx$.

73. Find $\int_{0}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin 2x} dx$.

74. Find $\int \frac{x+2}{\sqrt{9x-x^2}}dx$.

75. Evaluate $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$.

76. Evaluate $\int_{\frac{-\pi}{6}}^{\frac{\pi}{2}} \left(\sin|x|+\cos|x|\right)dx$.

77. If $\frac{d}{dx}(F(x))=\frac{1}{e^x+1}$, then find $F(x)$ given that $F(0)=\log\frac{1}{2}$.

78. Find $\int \frac{2x+1}{\sqrt{6x+x^2}}dx$.

79. Find $\int \frac{3x-1}{\sqrt{x^2-4x}}dx$.

80. If $I_1=\int_{-\pi/4}^{\pi/4}\frac{dx}{1+\cos 2x}$ and $I_2=\int_{-1/2}^{1/2}|x|dx$, then show that $I_1-4I_2=0$.

81. Find $\int_0^1 x\sin^{-1}x\,dx$.

82. Find $\int_0^1 \log(1+x^2)dx$.

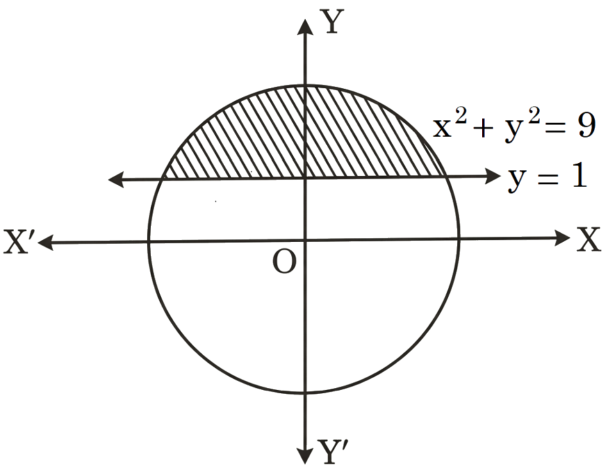
83. The area bounded by the curve $y=x|x|$, x-axis and the ordinates $x=-1$ and $x=1$ is given by

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

84. Which of the following expressions will give the area of region bounded by the curve $y=x^2$ and line $y=16$?

(a) $\int_0^4 x^2dx$ (b) $2\int_0^4 x^2dx$ (c) $\int_0^{16}\sqrt{y}\,dy$ (d) $2\int_0^{16}\sqrt{y}\,dy$

85. The area of the shaded region of the circle given below is equal to



(a) $\int_1^3\sqrt{9-y^2}dy$ (b) $2\int_1^3\sqrt{9-y^2}dy$ (c) $\int_0^3\sqrt{9-x^2}dx$ (d) $2\int_0^3\sqrt{9-x^2}dx$

86. The area of the region bounded by the curve $y=x$ and x-axis, between $x=0$ and $x=2$ is

(a) 2 sq. units (b) $\frac{1}{2}$ sq. unit (c) 1 sq. unit (d) 4 sq. units

87. An ant is observed crawling on a sheet of paper along a straight line given by equation $y=2x-4$. Area of the surface covered by the ant bounded by y-axis, x-axis and $x=1$ is

(a) 1 sq. unit (b) 3 sq. units (c) 2 sq. units (d) 4 sq. units

88. Using integration, find the area of the region enclosed by the curve $y=|x-6|$, the x-axis, and between $x=4$ and $x=8$.

89. Using integration, find the area of the region bounded by the curve $y=x|x|$, x-axis, $x=-2$ and $x=2$.

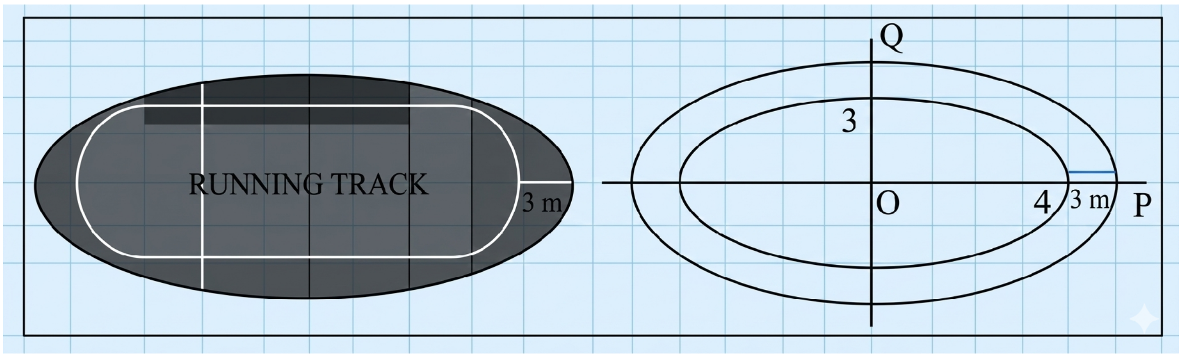
90. Using integration, find the area of the region bounded by $y=5x+4$, $y=0$, $x=-1$ and $x=1$.

91. Sketch the curve $\{(x,y):100x^2+25y^2=2500\}$ and find the area of the region enclosed by it, using integration.

92. Sketch the curve described by $\{(x,y):9x^2+16y^2=144\}$ and find the area of the region enclosed by it, using integration.

93. Sketch the graph defined by $\left\{(x,y):\frac{x^2}{25}+\frac{y^2}{25}=1\right\}$. Find the area of the region of minor segment cut off by the line $x=\frac{5}{2}$, using integration.

94. A racing track is build around an elliptical ground whose equation is given by $9x^2+16y^2=144$. The width of the track is 3 m as shown below.



Based on the given information, answer the following questions.
(i) Express y as a function of x from the given equation of ellipse.
(ii) Integrate the function obtained in (i) with respect to x.
(iii) (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration.

OR

(iii) (b) Write the coordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration.

95. Roundabouts are often made on busy roads to ease the traffic and avoid red lights.

One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$.

There is circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

Based on the given information, answer the following questions.

(i) Represent the given equations C_1 and C_2 with the help of a diagram.

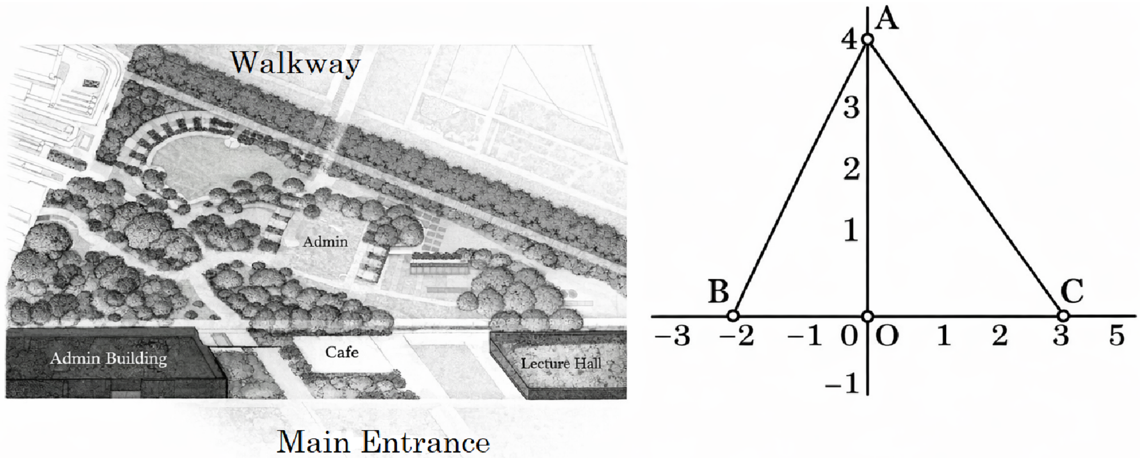
(ii) Express y as a function of x, ($y = f(x)$), for both C_1 and C_2.

(iii) (a) Using integration, find the area of region covered by the roundabout.

OR

(iii) (b) Using integration, find the area of region covered by circular pond.

96. There is a triangular park in the society. The park is divided into two sections as shown in the figure.



In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed. The vertices of the triangular park ABC are A(0, 4), B(–2, 0) and C(3, 0).

Based on the above information, answer the following questions.

(i) Write the equation of the boundary line AB of the park.

(ii) Write the equation of the boundary line AC of the park.

(iii) (a) Using integration, find the area of region OAC, in which children are allowed to play cricket, football.

OR

(iii) (b) Using integration, find the area of region AOB.

97. The integrating factor of differential equation $R\dfrac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is

(a) $e^{\int \frac{P}{Q} dy}$ (b) $e^{\int P dy}$ (c) $e^{\int \frac{P}{R} dy}$ (d) $e^{\int \frac{P}{R} dx}$

98. The order and degree of the differential equation $\dfrac{d}{dx}(e^y) = 0$ respectively are

(a) 0, 1 (b) 1, 1 (c) 2, 1 (d) 1, not defined

99. The order and degree of the differential equation $\dfrac{d}{dx}(\sin y) = y^2$ respectively are

(a) 1, 1 (b) 2, 1 (c) 2, 2 (d) 1, 2

100. The order and degree of the differential equation

$\dfrac{d}{dx}(y')^3 + (y')^3 = 1$ respectively are, where $y' = \dfrac{dy}{dx}$

(a) 1, 3 (b) 2, 1 (c) 3, 1 (d) 3, 2

101. $\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions?

(i) $F(x, y) = 3x + 2y$ (ii) $F(x, y) = \sin\frac{y}{x} + \log y - \log x$

(iii) $F(x, y) = e^{y/x} + 1$ (iv) $F(x, y) = \sqrt{x^2 + y^2} - y$

(a) (i) and (ii) (b) (i), (ii) and (iii) (c) (ii), (iii) and (iv) (d) (ii) and (iii)

102. Which of the following is **not** a Linear Differential Equation?

(a) $(1 + x^2)dy + 2xy\,dx = \cot x\,dx$ (b) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

(c) $x(1 + y^2)dx - y(1 + x^2)dy = 0$ (d) $y\,dx - (x + 3y^2)dy = 0$

103. The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is

(a) $3e^y = e^{3x} + C$ (b) $\log(3x - y) = C$ (c) $e^{3x-y} = C$ (d) $-e^y + 3e^{3x} = C$

***Direction :** Question given below is Assertion (A) and Reason (R) based question carrying **1 mark**. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

104. **Assertion (A) :** One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$.

Reason (R) : $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

105. Find the general solution of the differential equation $(y^2 - x^2)dx = 2xydy$.

106. Find the particular solution of the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$, given that $y(1) = 0$.

107. Find the general solution of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$.

108. Find the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$.

109. Find the particular solution of the differential equation $xy\frac{dy}{dx} = (x + 2)(y + 2)$, given that $y(1) = -1$.

110. Find a particular solution of the differential equation $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1$, given that $y = 0$ when $x = 0$.

111. Solve the differential equation $y\,e^y dx = (y^3 + 2x\,e^y)\,dy$ when $y(0) = 1$.

112. Solve the differential equation $(x - \sin y)\,dy + \tan y\,dx = 0$.

Unit IV - *Vector & 3 D Geometry*

Vector Algebra; Three Dimensional Geometry

01. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is

(a) 8 (b) –8 (c) 2 (d) $\frac{5}{2}$

02. If $|\vec{a}|=8$, $|\vec{b}|=3$ and $|\vec{a}\times\vec{b}|=12$, then the value of $|\vec{a}.\vec{b}|=$

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) $3\sqrt{12}$

03. If $|\vec{a}|=5$ and $-2\le\lambda\le1$, then the sum of greatest and the smallest value of $|\lambda\vec{a}|$ is

(a) –5 (b) 5 (c) 10 (d) 15

04. Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is

(a) $\hat{i}+2\sqrt{2}\,\hat{j}$ (b) $3\hat{k}$ (c) $\frac{3\sqrt{2}}{2}\hat{i}+\frac{3\sqrt{2}}{2}\hat{j}$ (d) $\sqrt{3}\,\hat{i}+\sqrt{3}\,\hat{j}+\sqrt{3}\,\hat{k}$

05. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?

(a) $\vec{a}.\vec{b}\le|\vec{a}||\vec{b}|$ (b) $|\vec{a}+\vec{b}|\ge|\vec{a}|+|\vec{b}|$ (c) $|\vec{a}-\vec{b}|=|\vec{a}|-|\vec{b}|$ (d) $|\vec{a}\times\vec{b}|\ge|\vec{a}||\vec{b}|$

06. If $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=198$ and $|\vec{a}|=10|\vec{b}|$, then

(a) $|\vec{a}|=\sqrt{2}$ (b) $|\vec{b}|=\sqrt{2}$ (c) $|\vec{b}|=10\sqrt{2}$ (d) $|\vec{a}|=\frac{10}{\sqrt{2}}$

07. If position vector $\vec{p}$ of a point (24, n) is such that $|\vec{p}|=25$, then the value of n is

(a) ±49 (b) ±5 (c) ±1 (d) ±7

08. If vectors $\vec{a}=3\hat{i}+2\hat{j}+\lambda\hat{k}$ and $\vec{b}=2\hat{i}-4\hat{j}+5\hat{k}$ represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is

(a) 1 (b) $\frac{5}{2}$

(c) $\frac{2}{5}$ (d) 0

Direction : *Questions given below are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
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(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

09. For two vectors $\vec{a}$ and $\vec{b}$, let θ be the angle between them.

Assertion (A) : $|\vec{a}\times\vec{b}|^2+(\vec{a}.\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$.

Reason (R) : $|\vec{a}\times\vec{b}|\tan\theta=(\vec{a}.\vec{b}),\ \left(\theta\ne\frac{\pi}{2}\right)$.

10. **Assertion (A) :** The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a}\ne\vec{0}$ are collinear vectors.

Reason (R) : $\vec{a}.(-2\vec{a})=0$.

11. Find the vector of magnitude 14 in the direction of $\overrightarrow{QP}$, where P and Q are the points (1, 3, 2) and (–1, 0, 8) respectively.

12. Vectors $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.

13. A vector $\vec{a}$ of magnitude 14 has direction ratios $<2,3,-6>$. Find the projection of the vector $\vec{a}$ on $\hat{i}$.

14. Using vectors, find the area of ΔABC with vertices A(1, 2, 3), B(2, –1,4) and C(4, 5, –1).

15. If $\overrightarrow{AB} = \hat{j} + \hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$ represent the two vectors along the sides AB and AC of ΔABC, prove that the median $\overrightarrow{AD} = \dfrac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$, where D is midpoint of BC. Hence, find the length of median AD.

16. If for two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

17. If the position vectors of three points A, B and C are $3\hat{i} + \hat{j}$, $5\hat{i} + 6\hat{j} - 3\hat{k}$ and $4\hat{j}$ respectively, then show that they form an isosceles triangle.

18. Let two rods placed on the ground be represented by vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.

19. A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.

20. If in a parallelogram PQRS, $\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\overrightarrow{QR} = \hat{i} - 2\hat{j} + \hat{k}$, then find the unit vectors parallel to the diagonals $\overrightarrow{PR}$ and $\overrightarrow{SQ}$.

21. If $(2\hat{i} + 3\hat{j} + \hat{k})$ and $(2\hat{i} + \hat{j} - \hat{k})$ represent the sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of ΔABC, find the vector representing the median through A.

22. Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$.

23. Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively, find the value of 'a'.

24. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \le 9$.

25. The length of perpendicular drawn from (2, 5, 7) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is

    (a) 0 (b) 5 (c) $\sqrt{74}$ (d) $\sqrt{78}$

26. The length of perpendicular drawn from the point (1, 2, 3) on line $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$ is

    (a) 0 (b) 6 (c) $\sqrt{10}$ (d) $\sqrt{14}$

27. The length of perpendicular drawn from the point (3, 4, 2) on the line $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ is

    (a) 2 (b) 9 (c) 5 (d) $\sqrt{29}$

28. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then

    (a) $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ (b) $\sin\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

    (c) $\tan\theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$ (d) $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

29. Direction ratios of lines l_1 and l_2 are $< 12, -3, 9 >$ and $< 4, q, -p >$ respectively. The values of p and q for which l_1 and l_2 are parallel are respectively

    (a) –1, 3 (b) 3, 1 (c) –3, –1 (d) –1, –3

30. Direction ratios of lines l_1 and l_2 respectively are $<1, -2, 3>$ and $<-2, p, -6>$. The values of p and q for which $l_1 \parallel l_2$ is
(a) –4 (b) 4 (c) –10 (d) 10

31. Direction ratios of lines l_1 and l_2 respectively are $<1, 0, 0>$ and $<0, -1, 0>$. The direction ratios of the line perpendicular to both l_1 and l_2 is
(a) $<1, 1, 0>$ (b) $<0, 0, -1>$ (c) $<1, 1, 1>$ (d) $<1, 0, -1>$

Direction : *Questions given below are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
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(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

32. **Assertion (A) :** Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$.
Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular to each other if $\vec{b}_1 . \vec{b}_2 = 0$.

33. **Assertion (A) :** A line can have direction cosines $<1, 1, 1>$.
Reason (R) : $\cos\theta = 1$ is possible for $\theta = 0$.

34. Find the coordinates of the point on the line $\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ such that the sum of coordinates is 3.

35. Find the coordinates of foot of perpendicular drawn from (0, 0, 0) to line $\frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2}$.

36. Find the coordinates of the point on line $x = \frac{y-1}{2} = \frac{z-2}{3}$ whose y coordinate is 3 times the x coordinate.

37. Find the angle between the following pair of lines $\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6}$ and $\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$.

38. If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value (s) of p.

39. Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$.

40. A line passing through the points A(1, 2, 3) and B(5, 8, 11) intersects the line $\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k})$. Find the coordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.

41. Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting

42. Prove that the line through points A(0, –1, –1) and B(4, 5, 1) intersects the line through points C(3, 9, 4) and D(–4, 4, 4). Hence, write the equation of line passing through the point of intersection of lines AB and CD as well as origin.

43. Show that line AB passing through points A(0, 4, 1), B(2, 3, –1) and the line CD passing through points C(4, 5, 0), D(2, 6, 2) are parallel. Also, find distance between them.

44. Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not.
$l_1 : \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$; $l_2 : \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$

45. Opposite sides of a square are along the lines :
$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$; $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$
Find the area of the square. If direction ratios of other pairs of opposite sides of the square are given by $< -3, 6, p >$, find the value of p.

46. Find a point on the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point $(1, 2, 3)$.

47. Find the equation of a line (in vector and Cartesian form) that passes through the point of intersection of lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ and is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

48. Find the vector and Cartesian equations of the line passing through the point of intersection of the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ and parallel to the line $\frac{x-1}{-2} = \frac{7-y}{-3} = z$.

49. Find the length of the perpendicular drawn from the point P(1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the equation of the perpendicular line joining P and the foot of the perpendicular.

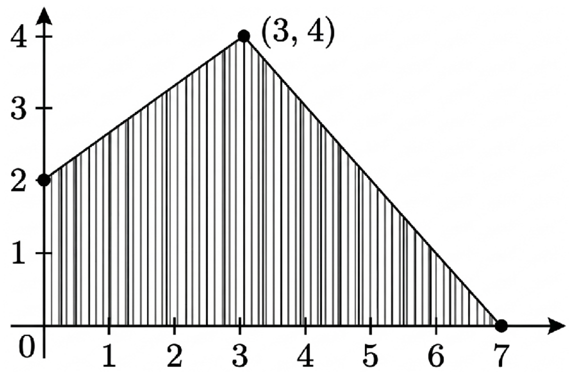
50. Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular.

51. Find the value of p if the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is $\frac{3}{\sqrt{2}}$ units.

Unit V - *Linear Programming*

Linear Programming

01. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below.



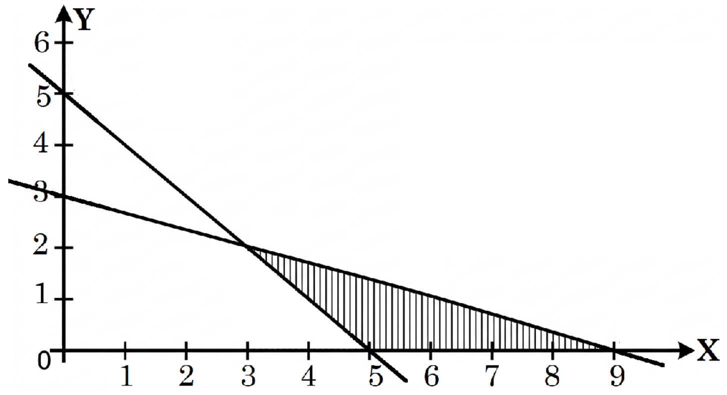
The maximum value of Z – minimum value of Z is

(a) 8 (b) 29 (c) 35 (d) 43

02. The degree of an objective function of a linear programming problem is

(a) 0 (b) 1 (c) 2 (d) Any natural number

03. For the feasible region shown below, the non-trivial constraints of the linear programming problem are



(a) $x+y\le 5,\ x+3y\le 9$ (b) $x+y\le 5,\ x+3y\ge 9$

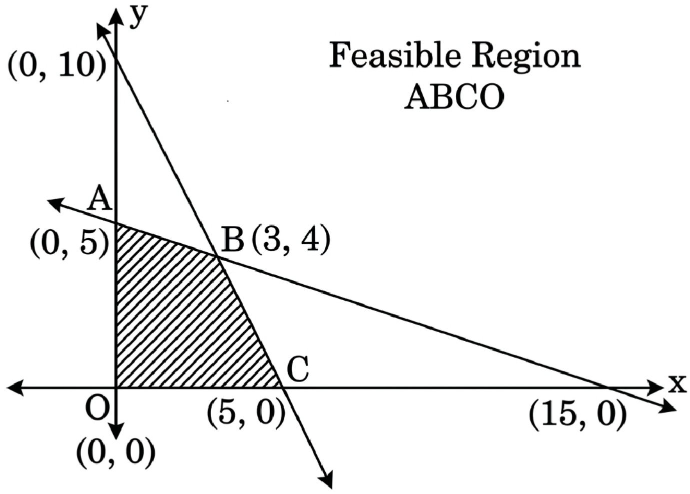
(c) $x+y\ge 5,\ x+3y\le 9$ (d) $x+y\ge 5,\ 3x+y\le 9$

04. The region represented by the system of inequations $3x+y\ge 3,\ 2x-y\ge -5;\ x,\ y\ge 0$ is

(a) unbounded in 1st quadrant (b) bounded in 1st quadrant

(c) unbounded in 2nd quadrant (d) bounded in 2nd quadrant

05. In the graph, the feasible region representing the Linear Programming Problem for maximizing objective function $Z=px+qy;\ p,\ q>0$ is shaded. If all points on segment AB give max (Z), then which of the following is true?



(a) $p=2q$ (b) $p=3q$ (c) $q=3p$ (d) $q=2p$

***Direction :** Question given below is Assertion (A) and Reason (R) based question carrying **1 mark**. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

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06. **Assertion (A) :** Consider a Linear Programming Problem with minimize $Z=x+2y$ subject to constraints $2x+y\ge 3,\ x+2y\ge 6;\ x,\ y\ge 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are (0, 3) and (6, 0).

Reason (R) : If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.

07. Solve the following linear programming problem graphically.

Minimize $Z = 13x - 15y$

Subject to constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0, y \ge 0$.

08. Solve the following Linear Programming Problem graphically.

Maximise $Z = 600x + 400y$

Subject to the constraints $x + 2y \le 12$, $4x + 5y \ge 20$, $2x + y \le 12$; $x, y \ge 0$.

09. Solve the following Linear Programming Problem graphically.

Maximise $Z = 12x + 18y$

Subject to the constraints $x + y \le 1200$, $x - 2y \ge 0$, $x + 3y \ge 600$, $x \ge 0$, $y \ge 0$.

10. Solve the following Linear Programming Problem graphically.

Maximise $Z = 200x + 120y$

Subject to the constraints $x + y \le 300$, $3x + y \le 600$, $x - y \ge -100$; $x, y \ge 0$.

11. Solve the following Linear Programming Problem graphically.

Maximise $Z = \frac{2x}{5} + \frac{3y}{10}$

Subject to the constraints $2x + y \le 1000$, $x + y \le 800$; $x, y \ge 0$.

Unit VI - Probability

Probability

01. For two events A and B such that $P(A) \ne 0$ and $P(B) \ne 1$, $P(A'|B') =$

(a) $1 - P(A|B)$ (b) $1 - P(A'|B)$ (c) $\frac{1 - P(A \cap B)}{P(B')}$ (d) $\frac{1 - P(A \cup B)}{P(B')}$

02. If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E|F) - P(F|E)$ is equal to

(a) $\frac{2}{7}$ (b) $\frac{3}{35}$ (c) $\frac{1}{70}$ (d) $\frac{1}{7}$

03. A box contains 4 red, 5 blue and 1 green marble. A child randomly takes out a marble from the box, notes down the colour and puts it back in the box. If the activity is repeated 3 times, what is the probability that at least one marble is red?

(a) $\frac{27}{125}$ (b) $\frac{8}{125}$ (c) $\frac{2}{125}$ (d) $\frac{98}{125}$

04. The probability that it will rain tomorrow in cities A, B and C is 60%, 70% and 80% respectively. The probability that it will rain tomorrow in at least one of the cities is

(a) $\frac{3}{250}$ (b) $\frac{244}{250}$ (c) 1 (d) $\frac{9}{10}$

05. The probability that a particular item is available in three shops A, B and C is $\frac{4}{5}, \frac{3}{4}$ and $\frac{2}{3}$ respectively. If a person visits all the three shops to buy the item, then what is the probability that it will be available in at least one of the shops?

(a) $\frac{59}{60}$ (b) 1 (c) $\frac{1}{60}$ (d) $\frac{9}{60}$

06. If $3P(A) = P(B) = \frac{3}{5}$ and $P(A|B) = \frac{1}{4}$, then $P(A \cup B)$ is

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{2}{15}$ (d) $\frac{13}{20}$

Direction : *Question given below is Assertion (A) and Reason (R) based question carrying* ***1 mark****. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

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(d) Assertion (A) is false but Reason (R) is true.

07. **Assertion (A) :** In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$.

Reason (R) : For any two events A and B, $P(A|B) = \frac{P(A \cup B)}{P(B)}$.

08. Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.

09. The probability of simultaneous occurrence of at least one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$.

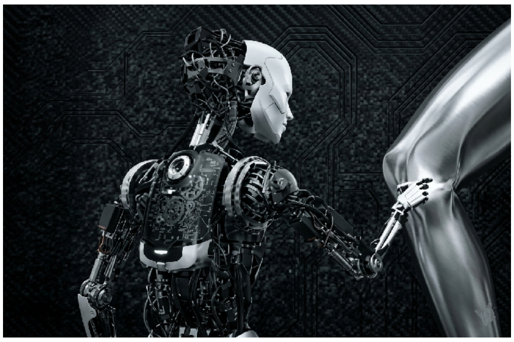
10. The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed.

The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

(i) target is hit. (ii) atleast one shot misses the target.

11. Mother, Father and Son line up at random for a family picture. Let event E : Son on one end and F : Father in the middle. Find $P(E|F)$.

12. A survey was conducted on the patients who have undergone knee replacement surgeries.



It was found that, Robotic Knee replacement surgeries have 90% success rate.
On a particular day, robotic surgery was performed on three patients, A, B and C, one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that

(i) exactly one surgery is successful. (ii) at most two surgeries are successful.

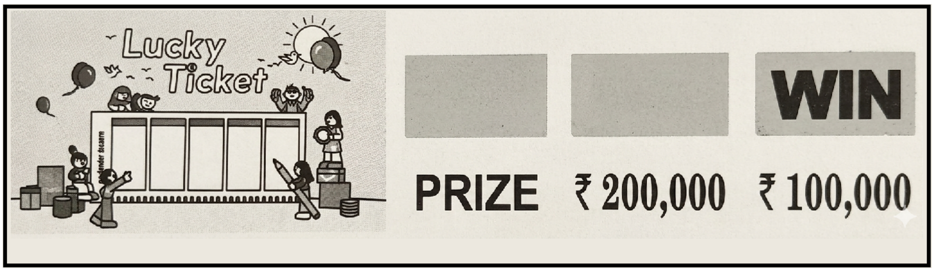
13. In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.

14. A die is rolled. Consider events : $A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$ and hence find

(i) $P(A|C)$ and $P(C|A)$ (ii) $P(A \cap B|C)$ and $P(A \cup B|C)$

15. A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B, the event of a number other than 4 on the first card selected. Find P(A and B) and find whether the events A and B are independent events or not.

16. In an online jackpot, there is one first prize of ₹300000, two second prizes of ₹200000 each and three third prizes of ₹50000 each.

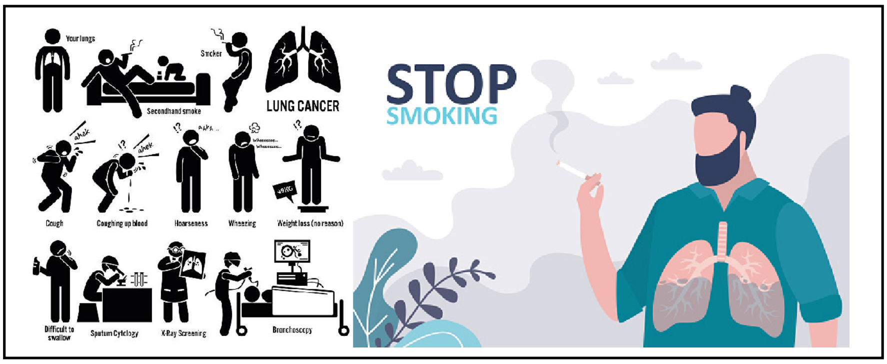


A total of 100000 jackpot tickets each costing ₹100 were sold there by raising a fund of ₹10000000.
Rohan bought one ticket.
Based on the given information, answer the following questions.
(i) What are the possible amounts, the person can win?
(ii) (a) What is the probability that the person wins at least ₹200000?

OR

(ii) (b) What is the probability that the person does not win any amount?
(iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹500000. The chances of winning the jackpot are 1 in 100000. Find the probability that on exactly one of tickets he wins the jackpot.

17. Smoking increases the risk of lung problems.

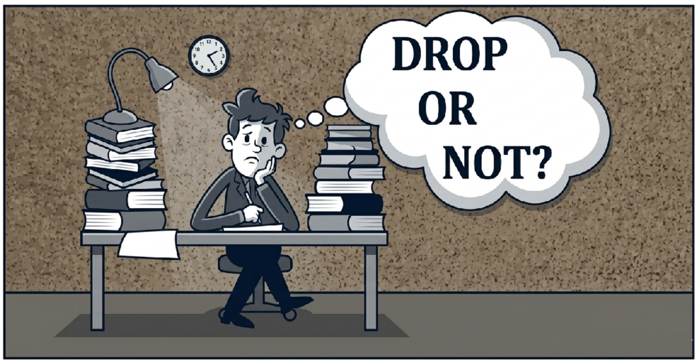


A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males.
A person is selected at random from these 50 people and tested for lung related problems.
Based on the given information, answer the following questions.
(i) What is the probability that selected person is a female?
(ii) If a male person is selected, what is the probability that he will not be suffering from lung problems?
(iii) (a) A person selected at random is detected with lung complications. Find the probability that selected person is a female.

OR

(iii) (b) A person selected at random is not having lung problems, find the probability that the person is a male.

18. A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.



As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII.
Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination.
Based on the given information, answer the following questions.
(i) Find the probability that a student selected at random is a regular student.
(ii) A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination?
(iii) (a) A student selected at random qualified the examination. Find the probability that student is not a dropout.

OR

(iii) (b) A student selected at random did not qualify the examination. Find the probability that the student was a regular student.

19. An NGO organizes a charity event in which they decide to distribute woolen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, and Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i = 1, 2, 3$.

Based on the above information, answer the following questions.
A person selects a cap.
(i) What is the probability that he selects a red cap?
(ii) If he selects a green cap, what is the probability that the cap has come from Box II?

20. There are three types of vaccines A_1, A_2, A_3, available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found

that 25% of the population was given Vaccine A_1, 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3. The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively.

Based on the above information, answer the following questions.
Find the probability that

(i) the person taking vaccine A_2 will get infected.

(ii) if a person is chosen randomly, he/she will be protected from the infection.

(iii) (a) the person was given Vaccine A_1, given that the randomly chosen person is infected.

OR

(iii) (b) the person was given Vaccine A_3, given that the randomly chosen person is not infected.

12 CLASS

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✔ Healthy Academic Interaction

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- Teachers are requested NOT to join student groups.
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☑ If you are already a member of any of our existing groups, please avoid joining another group to prevent repeated notifications of the same resources.
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With Regards
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Founder & Mentor
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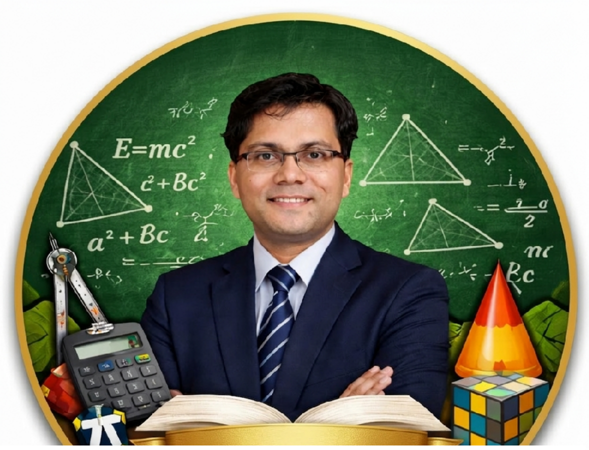

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